

$U(1)$ Chiral Symmetry in One-Dimensional Interacting Electron System with Spin

Taejin Lee

Department of Physics, Kangwon National University
Chuncheon 200-701 Korea

email: taejin@kangwon.ac.kr

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Astract

We study a spin dependent Tomonaga-Luttinger model in one dimension, which describes electron transport through a single barrier. Using the Fermi-Bose equivalence in one dimension, we map the model onto a massless Thirring model with a boundary interaction. A field theoretical perturbation theory for the model has been developed and the chiral symmetry is found to play an important role. The classical bulk action possesses a global $U_A(1)^4$ chiral symmetry, since the fermion fields are massless. This global chiral symmetry is broken by the boundary interaction and the bosonic degrees of freedom, corresponding to the chiral phase transformation, become dynamical. They acquire an additional kinetic action from the fermion path integral measure and govern the critical behaviors of physical operators. On the critical line where the boundary interaction becomes marginal, they decouple from the fermi fields. Consequently the action reduces to the free field action, which contains only a fermion bilinear boundary mass term as an interaction term. By a renormalization group analysis, we obtain a new critical line, which differs from the previously known critical lines in the literature. The result of this work implies that the phase diagram of the one dimensional electron system may have a richer structure than previously known.

1 Introduction

The chiral symmetry [1, 2] has been a key ingredient to understand many important subjects in particle physics and mathematical physics such as the neutral pion decay [1], anomaly cancellation conditions in gauge theories [3, 4], and instanton physics [5, 6]. We may enlarge the scope of its applications to condensed matter physics, which offers an exciting playground for both theorists and experimentalists with various different backgrounds. The subject, to be discussed in this paper is the $U(1)$ chiral symmetry in a field theoretical model, which describes electron transport through a single barrier in $1 + 1$ dimensions.

The model is formulated initially in terms of boson fields, representing the charge and spin degrees of freedom, as a spin dependent Tomogana-Luttinger model [7, 8] with a single barrier. It is this boson model, which has been studied extensively in the literature [9, 10, 11]. By virtue of Fermi-Bose equivalence in one dimension, boson fields can be mapped into fermion fields and vice versa. So it is possible to discuss the same model in two different frameworks. Although the model in both frameworks are equivalent to each other, often some features of the model are seen more transparently in one framework than the other. The critical behaviors of the Tomonaga-Luttinger model, which we are about to discuss in this paper, may be the case. At the critical points the boundary interaction, which is the only interaction at the critical point, can be represented by a bilinear operator in terms of fermion fields. In the framework of fermion theory, it is easy to understand that the models are exactly soluble and all the radiative corrections vanish at the critical points. Thus, the fermion theory may offer more adequate framework to explore the critical behaviors of the one dimensional electron system.

The electron transport in one dimension through a single barrier may be described by the Luttinger liquid model with a periodic boundary potential [10]. The model is formally equivalent to a quantum dissipative system subject to a periodic potential of Caldeira and Leggett type [12], called the Schmid model [13, 14]. The electronic transport of the one dimensional system has a quite interesting feature since the model exhibits phase transitions just as the Schmid model does. Introducing one more boson field corresponding to the spin density wave, one can describe the transportation of the electron with spin [11]. The main purpose of this work is to develop a field theoretical perturbation theory for the model and to explore their critical behaviors in the scheme of perturbation theory. To this end we choose the fermion framework.

In order to fermionize the model we may introduce two more auxiliary boson fields, just as we introduce an auxiliary boson field to fermionize the Schmid model. These auxiliary boson fields satisfy the Dirichlet boundary condition, so that they do not appear on the boundary action. Then by the Fermi-Bose equivalence, the bulk action of the four boson fields is transcribed into the self-interacting massless Thirring action with four flavors and the boundary periodic potential into the boundary fermion mass term. Since the bulk action does not contain a fermion mass term, the bulk action is invariant under the global $U_A(1)^4$ chiral and $U_V(1)^4$ vector phase transformations. But $U_A(1)^4$ chiral symmetry is broken by the boundary interaction and corresponding boson degrees of freedom become dynamical. The role of the boson fields corresponding to the local $U_V(1)^4$ vector phases of

the fermi fields is insignificant. Since they are free boson fields, which do not couple to any physical operator, they can be safely removed from the action. In contrast, the boson fields, which correspond to the local $U(1)_A^4$ chiral phase play an important role. Their bulk action receives additional contribution from the path integral measure through the chiral anomaly and they couple to the fermi fields. The radiative corrections to the boundary interaction mainly arise from the interaction between these boson fields and the fermi fields of charge and spin degrees of freedom. It is these boson fields which govern the critical behaviors of physical operators. If the contributions of the chiral anomaly to their kinetic action were not taken into account correctly, the renormalization group analysis of physical operators would be erroneous. At the critical points the boson fields of the local chiral phase of the fermi fields, decouple from the fermi fields and the action for the fermi fields becomes a free field one with a bilinear boundary mass term only.

2 Chiral $U_A(1)$ Symmetry: Transport of Spinless Electrons

We begin with a simpler model of the transport of spinless electrons through a single barrier first. The model is described by a single boson field and a periodic boundary potential

$$S = \frac{\alpha}{4\pi} \int d\tau d\sigma (\partial_\tau \phi \partial_\tau \phi + \partial_\sigma \phi \partial_\sigma \phi) + \frac{V_0}{2\pi} \int d\tau (e^{i\phi} + e^{-i\phi}) \Big|_{\sigma=0}. \quad (1)$$

If we integrate out the boson field ϕ on the bulk, we get a non-local boundary action,

$$S = \frac{\eta}{4\pi} \int d\tau d\tau' \frac{(\phi(\tau) - \phi(\tau'))^2}{(\tau - \tau')^2} + \frac{V_0}{2\pi} \int d\tau (e^{i\phi} + e^{-i\phi}) \Big|_{\sigma=0}. \quad (2)$$

where $\eta = \alpha/(2\pi)$. This model is known as the Schmid model, which has been studied extensively in the literature [14, 15] since the seminal paper by Schmid [13]. The model depicts a quantum dissipative system of Caldeira and Leggett type subject to a periodic potential and the parameter η in Eq.(2) is the friction coefficient. This action can be interpreted also as the open string action subject to a boundary periodic potential. The parameter α in Eq.(1) then can be interpreted the inverse of the Regge slope α' , which is related to the tension T of the string as follows

$$T = \frac{1}{2\pi\alpha'} = \frac{\alpha}{2\pi}. \quad (3)$$

When $\alpha = 1$, the system becomes critical. The action at the critical point has been studied in great detail in string theory because the model is proposed to depict the decay of unstable D-branes. It is called the rolling tachyon [16, 17, 18, 19]. The critical point $\alpha = 1$ is also the self-dual point under the particle-kink duality [13].

2.1 Fermionization of the Model for Spinless Electrons

It is easy to understand why the system becomes critical at the point, $\alpha = 1$ if we fermionize the model. It is straightforward to transcribe the bulk boson action into a bulk fermion action by using the well-established Fermi-Bose equivalence. But we must elaborate further in order to fermionize the boundary interaction term : $e^{i\phi}$:. The fermi fields ψ_L and ψ_R are represented in terms of the boson fields as : $\eta_L e^{-i\sqrt{2}\phi_L}$: and : $\eta_R e^{i\sqrt{2}\phi_R}$: repectively where $\eta_{L/R}$ are Klein factors. At a glance we find that the boundary term : $e^{i\phi}$: cannot be written as a fermion bilinear operator like $\psi_L^\dagger \psi_R$. This problem can be resolved [20] by introducing an auxiliary boson field φ , which satisfies the Dirichlet condition $\varphi = 0$ at the boundary. Then the composite operator : $e^{i(\phi \pm \varphi)}$: has precisely the property that we want. Defining two boson fields $\Phi_1 = (\varphi + \phi)/\sqrt{2}$, $\Phi_2 = (\varphi - \phi)/\sqrt{2}$, we may write the composite operator : $e^{i(\phi \pm \varphi)}$: as : $e^{\pm i\sqrt{2}\Phi_i}$:, $i = 1, 2$, which can be represented as a fermion bilinear operator both in the bulk and on the boundary in a consistent way. Because of the Dirichlet boundary condition, $\varphi = 0$ on the boundary, the composite : $e^{i(\phi \pm \varphi)}$: coincides with the boundary potential, : $e^{i\phi}$: on the boundary.

In terms of two boson fields Φ_i , $i = 1, 2$, we may rewrite the action Eq.(1) as

$$S = \frac{\alpha}{4\pi} \int d\tau d\sigma \sum_i^2 \partial\Phi_i \partial\Phi_i + \frac{V_0}{4\pi} \int d\sigma \sum_i^2 \left(e^{i\sqrt{2}\Phi_i} + e^{-i\sqrt{2}\Phi_i} \right) \Big|_{\tau=0}. \quad (4)$$

It should be noted that in Eq.(4) the two dimensional space-time coordinates τ and σ are interchanged, since it is more convenient to work in the closed string picture. Hereafter we will work on the model defined in the closed string picture, where the boundary is the spatial line, $\tau = 0$. The auxiliary boson field φ is a free boson field in the bulk and vanishes on the boundary. It is completely decoupled from the physical degrees of freedoms and its role is to give the right conformal dimension to the boundary operator. The boson fields $\Phi_{iL/R}$, $i = 1, 2$ are mapped onto the fermion fields as

$$\psi_L^1 = e^{-\frac{\pi}{2}i(p_L^1 + 2p_L^2 + p_R^1 + 2p_R^2)} e^{-\sqrt{2}i\Phi_L^1}, \quad (5a)$$

$$\psi_L^2 = e^{-\frac{\pi}{2}i(p_L^2 + p_R^2)} e^{-\sqrt{2}i\Phi_L^2}, \quad (5b)$$

$$\psi_R^1 = e^{-\frac{\pi}{2}i(p_L^1 + 2p_L^2 + p_R^1 + 2p_R^2)} e^{\sqrt{2}i\Phi_R^1} \quad (5c)$$

$$\psi_R^2 = e^{-\frac{\pi}{2}i(p_L^2 + p_R^2)} e^{\sqrt{2}i\Phi_R^2}. \quad (5d)$$

Here we give an explicit expression of the Klein factors, which ensure the anti-commutation relation between the fermi fields. The Klein factors are not unique. One may find some other, yet equivalent representations of Klein factors.

At the critical point $\alpha = 1$, the bulk action in the fermion theory is the free fermion action with two flavors and the boundary periodic interaction terms are replaced by fermion boundary mass terms, which are only quadratic in the fermion fields.

$$S = \frac{1}{2\pi} \int d\tau d\sigma \sum_{i=1}^2 \bar{\psi}_i \gamma^\mu \partial_\mu \psi_i + \frac{V_0}{4\pi} \int d\sigma \sum_{i=1}^2 \bar{\psi}_i \psi_i \Big|_{\tau=0}, \quad (6)$$

where $\psi_i = (\psi_{iL}, \psi_{iR})^t$, $i = 1, 2$ and

$$\gamma^0 = \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \gamma^1 = \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \gamma^5 = -i\gamma^0\gamma^1 = \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (7)$$

The action of the fermion model Eq.(6) clearly shows that the model is exactly solvable and the boundary interaction terms do not receive any radiative correction at the critical point.

At the off-critical point where $\alpha \neq 1$, we may write the action as

$$\begin{aligned} S = & \frac{1}{4\pi} \int d\tau d\sigma \sum_i^2 \partial\Phi_i \partial\Phi_i + \frac{\alpha-1}{4\pi} \int d\tau d\sigma \sum_i^2 \partial\Phi_i \partial\Phi_i \\ & + \frac{V_0}{4\pi} \int d\tau \sum_i^2 \left(e^{i\sqrt{2}\Phi_i} + e^{-i\sqrt{2}\Phi_i} \right) \Big|_{\tau=0}, \end{aligned} \quad (8)$$

and treat the second term as a part of the interaction action. By the Fermi-Bose equivalence

$$j_i^0 = \bar{\psi}_i \gamma^0 \psi_i = \psi_{iL}^\dagger \psi_{iL} + \psi_{iR}^\dagger \psi_{iR} = \sqrt{2} \partial_\sigma \Phi_i, \quad (9a)$$

$$j_i^1 = \bar{\psi}_i \gamma^1 \psi_i = i\psi_{iL}^\dagger \psi_{iL} - i\psi_{iR}^\dagger \psi_{iR} = -\sqrt{2} \partial_\tau \Phi_i, \quad i = 1, 2, \quad (9b)$$

the second term is transcribed into the Thirring interaction [21] term in the fermion theory. Hence, the fermionized action is given by

$$S = \frac{1}{2\pi} \int d\tau d\sigma \sum_i^2 \left(\bar{\psi}_i \gamma^\mu \partial_\mu \psi_i + \frac{g}{4} j_i^\mu j_{i\mu} \right) + \frac{V_0}{4\pi} \int d\sigma \sum_i^2 \bar{\psi}_i \psi_i \Big|_{\tau=0} \quad (10)$$

where $g = \alpha - 1$.

Introducing Abelian vector fields $A_{i\mu}$, $i = 1, 2$, we may rewrite the fermion action as follows

$$S = \frac{1}{2\pi} \int d\tau d\sigma \sum_i^2 \left[\bar{\psi}_i \gamma^\mu (\partial_\mu + iA_{i\mu}) \psi_i + \frac{1}{g} A_{i\mu} A_i^\mu \right] + \frac{V_0}{4\pi} \int d\sigma \sum_i^2 \bar{\psi}_i \psi_i \Big|_{\tau=0}. \quad (11)$$

Since the Abelian vector fields in 1 + 1 dimensions are generally decomposed as

$$A_i^\mu = \epsilon^{\mu\nu} \partial_\nu \theta_i + \partial^\mu \chi_i, \quad i = 1, 2, \quad (12)$$

we may rewrite the bulk action also in the following form

$$S_{\text{bulk}} = \frac{1}{2\pi} \int d\tau d\sigma \sum_i^2 \left[\bar{\psi}_i \gamma^\mu (\partial_\mu + i\epsilon^{\mu\nu} \partial_\nu \theta_i + i\partial^\mu \chi_i) \psi_i + \frac{1}{g} (\partial\theta_i \partial\theta_i + \partial\chi_i \partial\chi_i) \right]. \quad (13)$$

The interactions between the boson fields, θ_i , χ_i and the fermion fields in the bulk action Eq.(13) are removed by a local $U_V(1)^2 \times U_A(1)^2$ phase transformation

$$\psi_i = e^{-i\gamma_5 \theta_i - i\chi_i} \psi_{i0}, \quad \bar{\psi}_i = \bar{\psi}_{i0} e^{-i\gamma_5 \theta_i + i\chi_i}. \quad (14)$$

Then the bulk action is transformed into a free field action

$$S_{\text{bulk}} = \frac{1}{2\pi} \int d\tau d\sigma \sum_{i=1}^2 \left[\bar{\psi}_{i0} \gamma^\mu \partial_\mu \psi_{i0} + \frac{1}{g} (\partial\theta_i \partial\theta_i + \partial\chi_i \partial\chi_i) \right]. \quad (15)$$

If the boson fields θ_i and χ_i , corresponding to the local chiral and vector phases of the fermion fields respectively, are not coupled to physical operators in the boundary action, we may drop them from the action.

The local $U_A(1)^2$ chiral symmetry is broken in two ways: It is broken by the boundary interaction since the boundary mass term transforms under the chiral phase transformation as

$$\sum_i \bar{\psi}_i \psi_i = \sum_i \bar{\psi}_{i0} e^{-2i\gamma_5 \theta_i} \psi_{i0}. \quad (16)$$

And it is also broken by the chiral anomaly, which is manifested as the non-invariance of the path integral measure under the chiral transformation [22, 23, 24]

$$D[\psi]D[\bar{\psi}] = D[\psi_0]D[\bar{\psi}_0] \exp \left[-\frac{1}{2\pi} \int d\tau d\sigma \sum_i (\partial\theta_i)^2 \right]. \quad (17)$$

It yields an additional kinetic action for θ_i in the phase transformed action Eq.(15) so that the action S is written as

$$\begin{aligned} S = & \frac{1}{2\pi} \int d\tau d\sigma \sum_{i=1}^2 \left[\bar{\psi}_i \gamma^\mu \partial_\mu \psi_i + \left(\frac{1+g}{g} \right) \partial\theta_i \partial\theta_i + \partial\chi_i \partial\chi_i \right] \\ & + \frac{V_0}{4\pi} \int d\sigma \sum_i \bar{\psi}_i e^{-2i\gamma_5 \theta_i} \psi_i \Big|_{\tau=0} \end{aligned} \quad (18)$$

where we omit the subscript 0 for the fermi fields for the sake of convenience. While the $U(1)$ chiral symmetry is broken, the $U(1)$ vector symmetry $U_V(1)^2$ is kept unbroken: The measure and the boundary interaction are invariant under the vector phase transformation. Since the vector phase transformation is realized for the boson fields χ_i as a translation and χ_i are not coupled to the physical fermion fields, we may drop them from the action.

The final step is to scale the boson fields θ_i as

$$\theta_i \rightarrow \kappa \theta_i, \quad \kappa = \sqrt{\frac{g}{2(1+g)}} = \sqrt{\frac{\alpha-1}{2\alpha}}, \quad i = 1, 2. \quad (19)$$

It brings us to the following action, which contains a boundary interaction only

$$S = \frac{1}{2\pi} \int d\tau d\sigma \sum_{i=1}^2 \left\{ \bar{\psi}_i \gamma^\mu \partial_\mu \psi_i + \frac{1}{2} (\partial\theta_i)^2 \right\} + \frac{V_0}{4\pi} \int d\sigma \sum_i \bar{\psi}_i e^{-2i\gamma_5 \kappa \theta_i} \psi_i \Big|_{\tau=0}. \quad (20)$$

We note that the obtained action has a considerable advantage, being compared to other forms of the action. The bulk action contains only the free field action for fermion fields and boson fields and all the non-trivial interactions are encoded in the boundary action. Therefore, we only need to deal with the correlation functions on the one dimensional boundary. The only role of the bulk action is to define the free fermion and boson Green's functions on the boundary. From Eq.(20) it is clear that in the limit of the critical point, $\kappa \rightarrow 0$ ($\alpha \rightarrow 0$), the coupling between the boson fields θ_i and the fermion fields ψ_i vanishes. Thus, θ_i , becoming free fields, can be dropped from the action and the action reduces to the free fermion action with a boundary mass. In the next section we shall calculate the radiative corrections to the boundary interaction term, using the obtained action Eq.(20).

2.2 Radiative Corrections to the Boundary Periodic Potential

Our discussion on the radiative corrections begin with an expansion of the boundary action in κ , which is a small parameter near the critical point

$$S_{\text{boundary}} = \frac{V_0}{4\pi} \int d\sigma \sum_i [\bar{\psi}_i \psi_i - 2i\bar{\psi}_i \gamma_5 \kappa \theta_i \psi_i - 2\bar{\psi}_{ia} (\gamma_5 \kappa \theta_i)^2 \psi_i + \dots]. \quad (21)$$

In order to calculate the radiate corrections to the boundary periodic potential we consider the fully interacting two-point function (Green's function) on the boundary [25]. The fully interacting Green's function in the bulk (on the cylinder) is defined as

$$\begin{aligned} \mathbf{G}_{(i|\alpha\beta)}(\tau_1 - \tau_2; \sigma_1 - \sigma_2) &= \langle 0 | T \psi_{i\alpha}(\tau_1, \sigma_1) \bar{\psi}_{i\beta}(\tau_2, \sigma_2) | 0 \rangle \\ &= \int D[\bar{\psi}, \psi] \psi_{i\alpha}(\tau_1, \sigma_1) \bar{\psi}_{i\beta}(\tau_2, \sigma_2) e^{-S_0 - S_{\text{boundary}}}, \end{aligned} \quad (22)$$

where

$$S_0 = \frac{1}{2\pi} \int d\tau d\sigma \sum_{i=1}^2 \left[\bar{\psi}_i \gamma^\mu \partial_\mu \psi_i + \frac{1}{2} (\partial \theta_i)^2 \right], \quad (23a)$$

$$S_{\text{boundary}} = \frac{V_0}{4\pi} \int d\sigma \sum_{i=1}^2 \bar{\psi}_i e^{-2i\gamma_5 \kappa \theta_i} \psi_i \Big|_{\tau=0}. \quad (23b)$$

And the fully interacting Green's function on the boundary $\mathbf{G}(\sigma_1 - \sigma_2)$ is defined as the bulk Green's function evaluated on the boundary

$$\mathbf{G}_{\alpha\beta}(\sigma_1 - \sigma_2) = \lim_{\tau_1, \tau_2 \rightarrow 0} \langle 0 | T \psi_\alpha(\tau_1, \sigma_1) \bar{\psi}_\beta(\tau_2, \sigma_2) | 0 \rangle. \quad (24)$$

Since the action is diagonal in the flavor index i , the correlation functions, to be calculated, do not depend on the flavor index i . Hereafter the flavor index i shall be omitted for notational convenience. We also leave out the coordinate τ , since only operators on the boundary concern us.

For the Fermi fields on a cylinder, two conditions are available. We may choose the anti-periodic condition

$$\psi(2\pi) = -\psi(0), \quad \bar{\psi}(2\pi) = -\bar{\psi}(0), \quad \xi(2\pi) = -\xi(0), \quad \bar{\xi}(2\pi) = -\bar{\xi}(0), \quad (25)$$

or the periodic condition

$$\psi(2\pi) = \psi(0), \quad \bar{\psi}(2\pi) = \bar{\psi}(0), \quad \xi(2\pi) = \xi(0), \quad \bar{\xi}(2\pi) = \bar{\xi}(0). \quad (26)$$

The former is called the Neveu-Schwarz (NS) sector where the normal modes are labeled by half-odd-integers, $n \in \mathbb{Z} + 1/2$ and the latter is called the Ramond (R) sector where the normal modes are labeled by integers, $n \in \mathbb{Z}$. Here we choose the NS sector only, because the physical vacuum belongs to the NS sector, of which vacuum does not carry non-vanishing momentum. The vacuum in the R sector carries a non-vanishing momentum.

The Green's function on the boundary at the lowest order is defined as the free Green's function evaluated on the boundary, $G(0; \sigma_1 - \sigma_2)$. The fermion Green's function on the boundary has two different Fourier decompositions as follows, depending the direction, along which the limit Eq.(24) is taken

$$G(0+; \sigma_1 - \sigma_2) = \sum_{n \in \mathbb{Z} + 1/2} \frac{1}{2\pi} \gamma^0 \begin{pmatrix} \theta(n) & 0 \\ 0 & \theta(-n) \end{pmatrix} e^{in(\sigma_1 - \sigma_2)}, \quad (27a)$$

$$G(0-; \sigma_1 - \sigma_2) = - \sum_{n \in \mathbb{Z} + 1/2} \frac{1}{2\pi} \gamma^0 \begin{pmatrix} \theta(-n) & 0 \\ 0 & \theta(n) \end{pmatrix} e^{in(\sigma_1 - \sigma_2)}, \quad (27b)$$

where $\theta(n)$ is the unit step function (or Heaviside step function),

$$\theta(n) = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}. \quad (28)$$

Although they yield the same Green's function $G(\sigma)$ of closed form given as

$$G(\sigma) = \gamma^0 \frac{i}{4\pi \sin(\sigma/2)} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (29)$$

we must be careful when choosing the Fourier decomposition in the calculations of correlation functions.

At tree level the Green's function on the boundary receives a correction from the boundary interaction

$$\begin{aligned} \mathbf{G}_{\alpha\beta}^{(0)}(\sigma_1 - \sigma_2) &= \frac{1}{2!} \left(\frac{V_0}{2} \right)^2 \left\langle \psi_\alpha(\sigma_1) \bar{\psi}_\beta(\sigma_2) \left(\int \frac{d\sigma}{2\pi} \bar{\psi} \psi \right)^2 \right\rangle^{(0)} \\ &= \frac{V_0^2}{4} \int \frac{d\sigma'}{2\pi} \int \frac{d\sigma''}{2\pi} \left(G(\sigma_1 - \sigma') G(\sigma' - \sigma'') G(\sigma'' - \sigma_2) \right)_{\alpha\beta}. \end{aligned} \quad (30)$$

Recall that we treat the boundary mass term as a part of interaction even though the boundary mass term is a bilinear operator in terms of the fermi fields. Defining the Fourier transformation of the Green's functions as

$$\mathbf{G}_{\alpha\beta}^{(0)}(\sigma) = \sum_{n \in \mathbb{Z}+1/2} \mathbf{G}_{\alpha\beta}^{(0)}[n] e^{in\sigma}, \quad G(\sigma) = \sum_{n \in \mathbb{Z}+1/2} G[n] e^{in\sigma}, \quad (31)$$

we find

$$\mathbf{G}^{(0)}[n] = \frac{V_0^2}{4} \frac{1}{(2\pi)^2} G[n]. \quad (32)$$

At the first order the one-loop correction to the Green's function is given as

$$\begin{aligned} \mathbf{G}_{\alpha\beta}^{(1)}(\sigma_1 - \sigma_2) &= \frac{V_0^2}{2!} \left\langle \psi_\alpha(\sigma_1) \bar{\psi}_\beta(\sigma_2) \left(\int \frac{d\sigma}{2\pi} \bar{\psi} \gamma_5 \kappa \theta \psi \right)^2 \right\rangle^{(0)} \\ &= V_0^2 \kappa^2 \int \frac{d\sigma'}{2\pi} \int \frac{d\sigma''}{2\pi} \\ &\quad \left(G(\sigma_1 - \sigma') \gamma^5 G(\sigma' - \sigma'') \gamma^5 G(\sigma'' - \sigma_2) \right)_{\alpha\beta} G_\theta(\sigma' - \sigma''). \end{aligned} \quad (33)$$

Here $G_\theta(\sigma' - \sigma'')$ is the Green's function of the boson field θ on the boundary, which defined as

$$G_\theta(\sigma' - \sigma'') = \langle \theta(0, \sigma') \theta(0, \sigma'') \rangle^{(0)} = \sum_{n \in \mathbb{Z}} \frac{1}{2|n|} e^{in(\sigma' - \sigma'')}. \quad (34)$$

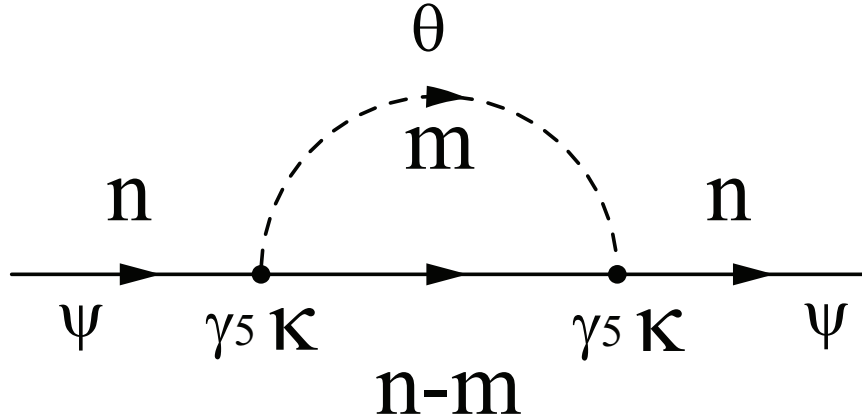


Figure 1: The first order correction to the Green's function of spinless Electron

The Feynman diagram Fig.1. depicts the evaluation of the first order correction to the Green's function of spinless electron. Since the Fourier transformation of a convolution

is the simple product of the Fourier transformed terms, the Fourier transformation of the first-order Green's function may be written as

$$\mathbf{G}^{(1)}[n] = V_0^2 \kappa^2 G[n] \left\{ \sum_{m \in \mathbb{Z}+1/2} \gamma^5 G[m] \gamma^5 G_\theta[n-m] \right\} G[n], \quad (35)$$

where $\mathbf{G}_{\alpha\beta}^{(1)}[n]$ and $G_\theta[n]$ are the Fourier transformation of the Green's functions defined as

$$\mathbf{G}_{\alpha\beta}^{(1)}(\sigma) = \sum_{n \in \mathbb{Z}+1/2} \mathbf{G}_{\alpha\beta}^{(1)}[n] e^{in\sigma}, \quad G_\theta(\sigma) = \sum_{n \in \mathbb{Z}} G_\theta[n] e^{in\sigma}. \quad (36)$$

By some algebra, we find

$$\mathbf{G}^{(1)}[n] = \frac{V_0^2 \kappa^2}{4\pi} \sum_m \frac{\gamma^0}{2\pi} \begin{pmatrix} \theta(n)\theta(-m) & 0 \\ 0 & \theta(-n)\theta(m) \end{pmatrix} \frac{1}{|n-m|}$$

This one-loop correction is divergent as expected. We must regularize it. Note that the two diagonal components of $\gamma^0 \mathbf{G}^{(1)}[n]$ are rewritten as

$$\begin{aligned} \sum_{m \in \mathbb{Z}+1/2} \theta(n)\theta(-m) \frac{1}{|n-m|} &= \theta(n) \sum_{m=1/2}^{\infty} \frac{1}{|n+m|} \\ &= \theta(n) \zeta(1) + \text{finite terms}, \end{aligned} \quad (37a)$$

$$\begin{aligned} \sum_{m \in \mathbb{Z}+1/2} \theta(-n)\theta(m) \frac{1}{|n-m|} &= \theta(-n) \sum_{m=1/2}^{\infty} \frac{1}{||n|+m|} \\ &= \theta(-n) \zeta(1) + \text{finite terms} \end{aligned} \quad (37b)$$

and they can be regularized by the Riemann zeta function $\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$,

$$\zeta(1) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots = \lim_{\Lambda \rightarrow \infty} (\gamma + 2 \ln \Lambda^2), \quad (38)$$

where γ is the EulerMascheroni constant, $\gamma = 0.57721566 \dots$. In passing we also note that this divergence corresponds to the ultraviolet divergence and the one-loop correction to the Green's function is free of infrared divergence.

Up to the one-loop order the radiative corrections to the Green's function are evaluated in the momentum number space as

$$\mathbf{G}^{(0)}[n] + \mathbf{G}^{(1)}[n] = \frac{V_0^2}{16\pi^2} \left(1 + \frac{\kappa^2}{2} \zeta(1) + \cdots \right) G[n]. \quad (39)$$

This divergence can be removed by renormalizing the coupling constant V as follows

$$V^2 = V_0^2 \left(1 + \kappa^2 \ln \frac{\Lambda^2}{\mu^2} \right). \quad (40)$$

This renormalization group flow defines the phase structure of the model,

$$V_0^2 \left(1 + \frac{\alpha - 1}{2\alpha} \ln \frac{\Lambda^2}{\mu^2} \right) = V_0^2 \left(\frac{\Lambda^2}{\mu^2} \right)^{\frac{\alpha-1}{2\alpha}}. \quad (41)$$

When $\alpha > 1$ the periodic potential becomes a relevant operator and when $\alpha < 1$ the potential becomes an irrelevant operator. Thus, in the region where $\alpha > 1$, the periodic potential is strong and the particles are mostly localized in the minima of the potential. In the other region where $\alpha < 1$, the potential is weak and particles are delocalized. The point $\alpha = 1$ sets the phase boundary. The higher order corrections may generate descendent perturbations of type $e^{in\phi}$, $n = 2, 3, \dots$, but they do not change the phase structure of the model. The perturbation analysis shows that the operator $e^{in\phi}$ becomes a relevant operator where $\alpha = n^2 > 1$. In this region the particles are already localized by the periodic potential due to the primary operator $e^{i\phi}$.

3 Chiral $U_A(1)$ Symmetry in Transport of Electrons with Spin

Being equipped with the field theoretical perturbation tool, developed in the last section, we will discuss the more complex system, namely the transport of one-dimensional electrons with spin. The one-dimensional single-channel interacting electron with spin system is described by the following action [11]

$$S = \frac{1}{4\pi} \int_0^{\beta_T} dt \int dx \left[\frac{1}{v_\rho \eta_\rho} (\partial_t \theta)^2 + \frac{v_\rho}{\eta_\rho} (\partial_x \theta)^2 + \frac{1}{v_\sigma \eta_\sigma} (\partial_t \phi)^2 + \frac{v_\sigma}{\eta_\sigma} (\partial_x \phi)^2 \right] + V'_0 \int_0^{\beta_T} dt \cos \theta \cos \phi \Big|_{x=0} \quad (42)$$

where the phase fields θ and ϕ represent charge and spin density fluctuations respectively and the boundary term represents the impurity (barrier) potential. We refer the reader to ref.[11] for parameters appearing in Eq.(42). We may map the model into a string theory action with a periodic tachyon potential on a disk

$$S = \frac{\alpha_\rho}{4\pi} \int d\tau d\sigma [(\partial_\tau \theta)^2 + (\partial_\sigma \theta)^2] + \frac{\alpha_\sigma}{4\pi} \int d\tau d\sigma [(\partial_\tau \phi)^2 + (\partial_\sigma \phi)^2] + 2V_0 \int \frac{d\sigma}{2\pi} \cos \theta \cos \phi \Big|_{\tau=0} \quad (43)$$

where

$$\sigma = \frac{2\pi}{\beta_T} t, \quad \alpha_\rho = \frac{1}{\eta_\rho}, \quad \alpha_\sigma = \frac{1}{\eta_\sigma}, \quad V_0 = \frac{\beta_T}{2} V'_0 \quad (44)$$

and

$$\theta(\tau, \sigma) = \theta\left(\frac{2\pi}{\beta_T} \frac{1}{v_\rho} x, \frac{2\pi}{\beta_T} t\right), \quad \phi(\tau, \sigma) = \phi\left(\frac{2\pi}{\beta_T} \frac{1}{v_\sigma} x, \frac{2\pi}{\beta_T} t\right) \quad (45)$$

Since the action is given as a direct sum of the actions for θ and ϕ , we scale x differently in their actions. (Note that the boundary interaction term does not depend on τ .)

It is convenient to rewrite the action in terms of scalar fields ϕ_a , $a = 1, 2$ defined as

$$\phi_1 = \theta + \phi, \quad \phi_2 = \theta - \phi. \quad (46)$$

Then the action is written as

$$S_\phi = \frac{1}{4\pi} \int d\tau d\sigma [\partial\phi_a \partial\phi_a + g^{ab} \partial\phi_a \partial\phi_b] + \frac{V_0}{2} \int \frac{d\sigma}{2\pi} \sum_{a=1}^2 (e^{i\phi_a} + e^{-i\phi_a}) \Big|_{\tau=0} \quad (47)$$

where $g^{11} = g^{22} = \frac{\alpha_\rho + \alpha_\sigma}{4} - 1$, $g^{12} = g^{21} = \frac{\alpha_\rho - \alpha_\sigma}{4}$. If $g^{ab} = 0$, or equivalently, $\alpha_\rho = \alpha_\sigma = 2$ ($\eta_\rho = \eta_\sigma = 1/2$), we see that the action reduces to a critical theory with a sum of two commuting marginal perturbations

$$S_\phi = \frac{1}{4\pi} \int d\tau d\sigma \partial\phi_a \partial\phi_a + \frac{V_0}{2} \int \frac{d\sigma}{2\pi} \sum_a (e^{i\phi_a} + e^{-i\phi_a}) \Big|_{\tau=0}. \quad (48)$$

So it is clear that the point $\alpha_\rho = \alpha_\sigma = 2$ ($\eta_\rho = \eta_\sigma = 1/2$) must be the critical point, where the model can be described by four Dirac fermion fields with a bilinear boundary mass.

In order to apply the field theoretical perturbation analysis to the model, developed in the previous section, we should fermionize the model first. The fermionization begins with introducing auxiliary free boson fields φ^a of which action has the same form as that for ϕ^a , $a = 1, 2$, except for the periodic boundary potential terms

$$S_\varphi = \frac{1}{4\pi} \int d\tau d\sigma [\partial\varphi_a \partial\varphi_a + g^{ab} \partial\varphi_a \partial\varphi_b]. \quad (49)$$

Here the Dirichlet boundary condition is chosen for φ_a . Since no boundary terms for φ_a are introduced and the bulk action for φ_a is the free field action, φ_a is decoupled from the physical fields. Their role is to give the boundary operators the right dimensions in the bulk and on the boundary, so that the boundary operators can be transcribed into bilinear fermion operators.

Using the Fermi-Bose equivalence, we may write the fermion fields in terms of the boson fields as

$$\begin{aligned} \psi_{a1L} &= \zeta_{a1L} : e^{-i\sqrt{2}\Phi_{a1L}} :, & \psi_{a2L} &= \zeta_{a2L} : e^{-i\sqrt{2}\Phi_{a2L}} : \\ \psi_{a1R} &= \zeta_{a1R} : e^{i\sqrt{2}\Phi_{a1R}} :, & \psi_{a2R} &= \zeta_{a2R} : e^{i\sqrt{2}\Phi_{a2R}} :, \end{aligned} \quad (50)$$

where

$$\Phi_{a1} = \frac{1}{\sqrt{2}} (\varphi_a + \phi_a), \quad \Phi_{a2} = \frac{1}{\sqrt{2}} (\varphi_a - \phi_a), \quad a = 1, 2, \quad (51)$$

and ζ_{aiL} , ζ_{aiR} are cocycles which ensure the anti-commutation relationship between the fermion fields. Accordingly, the action can be written in terms of the fermion fields as

$$\begin{aligned} S &= S_\phi + S_\varphi \\ &= \frac{1}{2\pi} \int d\tau d\sigma \left(\sum_{a,i} \bar{\psi}_{ai} \gamma^\mu \partial_\mu \psi_{ai} + \sum_{a,b,i} \frac{g^{ab}}{4} j^\mu_{ai} j_{\mu bi} \right) + \frac{V_0}{4\pi} \int d\sigma \sum_{a,i} \bar{\psi}_{ai} \psi_{ai} \Big|_{\tau=0} \end{aligned} \quad (52)$$

where $j_{\mu ai} = \bar{\psi}_{ai} \gamma^\mu \psi_{ai}$. This action can be understood as a four flavor Thirring model with boundary masses.

We may rewrite the Thirring action as follows, introducing four Abelian $U(1)$ vector fields $A_{i\mu a}$, $i, a = 1, 2$

$$S = \frac{1}{2\pi} \int d\tau d\sigma \sum_{a,i} [\bar{\psi}_{ai} \gamma^\mu (\partial_\mu + i A_{i\mu a}) \psi_{ai} + A_{i\mu a} (g^{-1})_{ab} A_{ib}^\mu] + \frac{V_0}{4\pi} \int d\sigma \sum_{a,i} \bar{\psi}_{ai} \psi_{ai} \Big|_{\tau=0}. \quad (53)$$

Decomposing the Abelian vector fields

$$A_{ia}^\mu = \epsilon^{\mu\nu} \partial_\nu \theta_{ia} + \partial^\mu \chi_{ia}, \quad i = 1, 2 \quad a = 1, 2. \quad (54)$$

we can remove the interaction between the vector fields and the fermion fields by a local phase transformation,

$$\psi_{ia} = e^{-i\gamma_5 \theta_{ia} - i\chi_{ia}} \psi_{ia0}, \quad \bar{\psi}_{ia} = \bar{\psi}_{ia0} e^{-i\gamma_5 \theta_{ia} + i\chi_{ia}}. \quad (55)$$

Under the local phase transformation the path integral measure for the fermi fields transforms as

$$D[\psi] D[\bar{\psi}] = D[\psi_0] D[\bar{\psi}_0] \exp \left[-\frac{1}{2\pi} \int d\tau d\sigma \sum_{i,a} (\partial \theta_{ia})^2 \right]. \quad (56)$$

After the local phase transformation, the bulk action contains only free field actions for four fermion fields and eight boson fields

$$S_{\text{bulk}} = \frac{1}{2\pi} \int d\tau d\sigma \sum_{i=1}^2 [\bar{\psi}_{ia0} \gamma^\mu \partial_\mu \psi_{ia0} + \partial \theta_{ia} (g^{-1} + I)_{ab} \partial \theta_{ib} + \partial \chi_{ia} (g^{-1})_{ab} \partial \chi_{ib}]. \quad (57)$$

The additional kinetic term for the boson fields θ_{ia} comes from the path integral measure for the fermi fields. This is the manifestation of the chiral $U_A(1)^4$ anomaly. The action is diagonal in the flavor index i . If the massless boson fields θ_{ia} and χ_{ia} do not couple to the fermion fields ψ_{ia} , we may drop the boson fields from the action, leaving the massless fermion fields only in the action. The global $U_A(1)^4$ symmetry is broken by the boundary interaction term

$$\sum_{i,a} \bar{\psi}_{ia} \psi_{ia} = \sum_{i,a} \bar{\psi}_{ia0} e^{-2i\gamma_5 \theta_{ia}} \psi_{ia0}, \quad (58)$$

and the corresponding degrees of freedom, the boson fields θ_{ia} couple to the fermi fields. On the other hand, the boundary action is invariant under the $U_V(1)^4$ phase transformation, and the corresponding boson fields χ_{ia} do not couple to the fermi fields. Thus, we may drop these free boson fields from the action.

We can diagonalize the matrix g_{ab}^{-1} by a similarity transformation

$$M^t g^{-1} M = \begin{pmatrix} \frac{2}{\alpha_\rho - 2} & 0 \\ 0 & \frac{2}{\alpha_\sigma - 2} \end{pmatrix}, \quad (59)$$

where

$$M = M^{-1} = M^t = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \quad M^t M = I. \quad (60)$$

Under the similarity transformation for $\theta_{ia} = M_{ab} \theta'_{ia}$,

$$g^{-1} + I \rightarrow M^{-1} g^{-1} M + I = \begin{pmatrix} \frac{\alpha_\rho}{\alpha_\rho - 2} & 0 \\ 0 & \frac{\alpha_\sigma}{\alpha_\sigma - 2} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1/\kappa_1^2 & 0 \\ 0 & 1/\kappa_2^2 \end{pmatrix}. \quad (61)$$

The bulk action for $\theta_{ia} = M_{ab} \theta'_{ib}$ becomes

$$S_\theta = \frac{1}{4\pi} \int d\tau d\sigma \sum_i \left[\kappa_1^{-2} (\partial \theta'_{i1})^2 + \kappa_2^{-2} (\partial \theta'_{i2})^2 \right]. \quad (62)$$

Then we scale θ' to have a free field action for θ in the bulk

$$\theta'_{i1} \rightarrow \kappa_1 \theta'_{i1}, \quad \theta'_{i2} \rightarrow \kappa_2 \theta'_{i2}. \quad (63)$$

where

$$\kappa_1 = \sqrt{\frac{\alpha_\rho - 2}{2\alpha_\rho}}, \quad \kappa_2 = \sqrt{\frac{\alpha_\sigma - 2}{2\alpha_\sigma}}. \quad (64)$$

The scaling is equivalent to the following transformation

$$\theta'_a = \mathbf{K}_{ab} \theta''_b, \quad \mathbf{K} = \begin{pmatrix} \kappa_1 & 0 \\ 0 & \kappa_2 \end{pmatrix} \quad (65)$$

The similarity transformation and the scaling bring us to the the boundary mass term

$$\sum_{i,a} \bar{\psi}_{ia} \psi_{ia} = \sum_{i,a} \bar{\psi}_{ia0} e^{-2i\gamma_5 (M\mathbf{K})_{ab} \theta''_{ib}} \psi_{ia0} \quad (66)$$

and the action in the desired form

$$S = \frac{1}{2\pi} \int_M d\tau d\sigma \sum_{i=1}^2 \left[\bar{\psi}_{ia} \gamma^\mu \partial_\mu \psi_{ia} + \frac{1}{2} \partial \theta_{ia} \partial \theta_{ia} \right] + \frac{V_0}{4\pi} \int d\sigma \sum_{i,a} \bar{\psi}_{ia} e^{-2i\gamma_5 (M\mathbf{K})_{ab} \theta_{ib}} \psi_{ia}. \quad (67)$$

Here we omit the subscript 0 for the fermi fields and the superscript $''$ for θ fields for the sake of convenience. Near the critical line, \mathbf{K} has small eigenvalues and the boundary action may be expanded in \mathbf{K}

$$\begin{aligned} S_{\text{boundary}} = \frac{V_0}{4\pi} \int d\sigma \sum_{i,a} & \left[\bar{\psi}_{ia} \psi_{ia} - 2i \bar{\psi}_{ia} \gamma_5 (M\mathbf{K})_{ab} \theta_{ib} \psi_{ia} \right. \\ & \left. - 2 \bar{\psi}_{ia} (\gamma_5 (M\mathbf{K})_{ab} \theta_{ib})^2 \psi_{ia} + \dots \right] \Big|_{\tau=0}. \end{aligned} \quad (68)$$

Now we are ready to calculate the radiative corrections to the boundary periodic potential in the scheme of perturbation theory of a renormalizable field theory.

4 Radiative Corrections to the Periodic Potential

Our discussion on the radiative corrections to the boundary periodic potential of the model of electrons with spin is parallel to the previous discussion on the radiative corrections in the model of spinless electrons. Corrections to the boundary mass follow from examining the fully interacting propagator

$$\begin{aligned} \mathbf{G}_{(ia|\alpha\beta)}(\sigma_1 - \sigma_2) &= \langle 0 | T \psi_{ia\alpha}(0, \sigma_1) \bar{\psi}_{ia\beta}(0, \sigma_2) | 0 \rangle \\ &= \int D[\bar{\psi}, \psi] \psi_{ia\alpha}(0, \sigma_1) \bar{\psi}_{ia\beta}(0, \sigma_2) \exp(-S_0 - S_{\text{boundary}}). \end{aligned} \quad (69)$$

We treat the boundary mass as a part of interaction as before. The first term in the expansion of the boundary action S_{boundary} , Eq.(68) contributes to the corrections of the Green's function at tree level $\mathbf{G}_{(ia|\alpha\beta)}^{(0)}$

$$\begin{aligned} \mathbf{G}_{(ia|\alpha\beta)}^{(0)}(\sigma_1 - \sigma_2) &= \frac{1}{2!} \frac{V_0^2}{4} \left\langle \psi_{ia\alpha}(0, \sigma_1) \bar{\psi}_{ia\beta}(0, \sigma_2) \left(\sum_b \int \frac{d\sigma}{2\pi} \bar{\psi}_{ib} \psi_{ib} \right)^2 \right\rangle_{\tau=0}^{(0)} \\ &= \frac{V_0^2}{4} \int \frac{d\sigma'}{2\pi} \int \frac{d\sigma''}{2\pi} (G(\sigma_1 - \sigma') G(\sigma' - \sigma'') G(\sigma'' - \sigma_2))_{\alpha\beta}. \end{aligned} \quad (70)$$

Since $\mathbf{G}_{(ia|\alpha\beta)}^{(0)}(\sigma)$ does not depend on the indices i and a , we omit these indices in the following calculations. The calculation of this tree level correction is just same as that for the model of spinless electron system Eqs.(31,32). The Fourier transformation of $\mathbf{G}_{\alpha\beta}^{(0)}(\sigma)$ is given by

$$\mathbf{G}_{\alpha\beta}^{(0)}[n] = \int \frac{d\sigma}{2\pi} \mathbf{G}_{\alpha\beta}^{(0)}(\sigma) e^{-in\sigma} = \frac{V_0^2}{16\pi^2} G_{\alpha\beta}[n], \quad n \in \mathbb{Z} + 1/2. \quad (71)$$

The first-order correction to the Green's function $\mathbf{G}_{(ia|\alpha\beta)}^{(1)}$ is obtained also in a similar way. The iteration of the second term in the expansion of the boundary action S_{boundary} , Eq.(68) leads to the first-order one-loop correction to the Green's function

$$\begin{aligned} \mathbf{G}_{(ia|\alpha\beta)}^{(1)}(\sigma_1 - \sigma_2) &= \frac{V_0^2}{2!} \left\langle \psi_{a\alpha}(0, \sigma_1) \bar{\psi}_{a\beta}(0, \sigma_2) \right. \\ &\quad \left. \left(\sum_{b,c} \int \frac{d\sigma}{2\pi} \bar{\psi}_b \gamma_5 (M\mathbf{K})_{bc} \theta_c \psi_b \right)^2 \right\rangle_{\tau=0}^{(0)} \\ &= \frac{V_0^2}{2} \text{Tr}((M\mathbf{K})(M\mathbf{K})^t) \int \frac{d\sigma'}{2\pi} \int \frac{d\sigma''}{2\pi} \\ &\quad \left(G(\sigma_1 - \sigma') \gamma^5 G(\sigma' - \sigma'') \gamma^5 G(\sigma'' - \sigma_2) \right)_{\alpha\beta} G_{\theta}(\sigma' - \sigma''). \end{aligned} \quad (72)$$

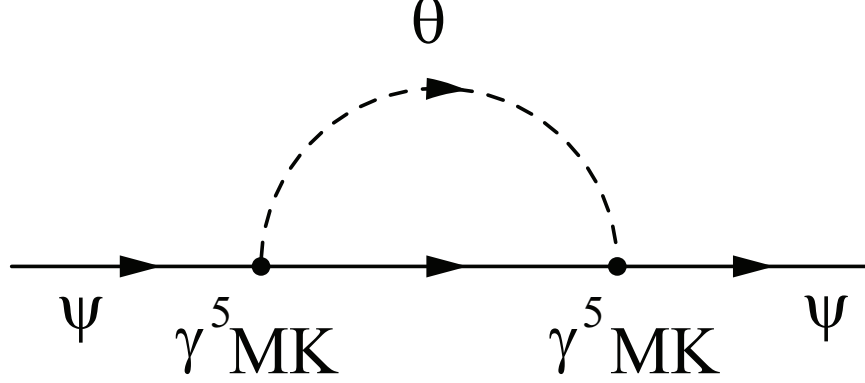


Figure 2: The first order correction to the Green's function of electron with spin.

Fig 2. depicts the Feynman diagram corresponding to the first order correction $\mathbf{G}_{(ia|\alpha\beta)}^{(1)}$. The first order correction $\mathbf{G}_{(ia|\alpha\beta)}^{(1)}$ is proportional to

$$\begin{aligned}
(\mathbf{MK}(\mathbf{MK})^t)_{11} &= (\mathbf{MK}(\mathbf{MK})^t)_{22} \\
&= \frac{1}{2}(\kappa_1^1 + \kappa_2^2) \\
&= \frac{1}{2} \left(1 - \frac{1}{\alpha_\rho} - \frac{1}{\alpha_\sigma} \right) \\
&= \frac{1}{2} (1 - \eta_\rho - \eta_\sigma).
\end{aligned} \tag{73}$$

The rest part of the calculation of $\mathbf{G}_{(ia|\alpha\beta)}^{(1)}$ is just same as that of the first order correction to the Green's function of spinless electrons, calculated in the previous section

$$\mathbf{G}_{\alpha\beta}^{(1)}[n] = \frac{V_0^2}{16\pi^2} \frac{1}{2} \left(1 - \frac{1}{\alpha_\rho} - \frac{1}{\alpha_\sigma} \right) (\zeta(1) + \text{finite terms}) G_{\alpha\beta}[n], \tag{74}$$

where $\mathbf{G}_{\alpha\beta}^{(1)}[n]$ is the Fourier transformation of $\mathbf{G}_{(ia|\alpha\beta)}^{(1)}$

$$\mathbf{G}_{\alpha\beta}^{(1)}[n] = \int \frac{d\sigma}{2\pi} \mathbf{G}_{\alpha\beta}^{(1)}(\sigma) e^{-in\sigma}, \quad n \in \mathbb{Z} + 1/2. \tag{75}$$

Up to the first order the radiative corrections to the Green's function is obtained in the momentum number space as

$$\mathbf{G}_{\alpha\beta}^{(0)}[n] + \mathbf{G}_{\alpha\beta}^{(1)}[n] = \frac{V_0^2}{16\pi^2} \left(1 + \frac{1}{2} \left(1 - \frac{1}{\alpha_\rho} - \frac{1}{\alpha_\sigma} \right) \zeta(1) + \dots \right) G_{\alpha\beta}[n] \tag{76}$$

As in the case of the spinless electron, we can remove this ultraviolet divergence by renormalizing the coupling constant V

$$\begin{aligned} V^2 &= V_0^2 \left(1 + \left(1 - \frac{1}{\alpha_\rho} - \frac{1}{\alpha_\sigma} \right) \ln \frac{\Lambda^2}{\mu^2} \right) \\ &= V_0^2 \left(\frac{\Lambda^2}{\mu^2} \right)^{1 - \frac{1}{\alpha_\rho} - \frac{1}{\alpha_\sigma}}. \end{aligned} \quad (77)$$

The structure of the phase diagram is fixed by the renormalization group flow. Where $1 - \frac{1}{\alpha_\rho} - \frac{1}{\alpha_\sigma} > 0$ (equivalently $1 - \eta_\rho - \eta_\sigma > 0$, the region I in the figure 3) the periodic potential becomes a relevant operator and the potential tends to be strong in the zero temperature limit. In the region II of figure 3 where $1 - \frac{1}{\alpha_\rho} - \frac{1}{\alpha_\sigma} < 0$ (equivalently $1 - \eta_\rho - \eta_\sigma < 0$), the periodic potential becomes an irrelevant operator and the potential is weak in the zero temperature limit.

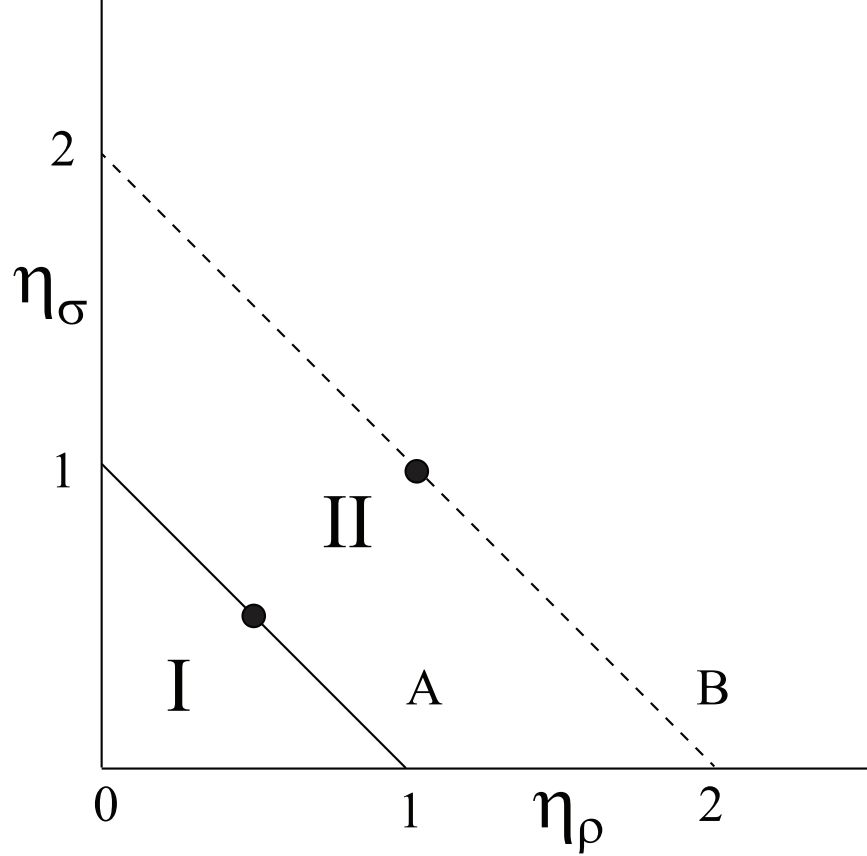


Figure 3: The phase diagram and the critical Line, $\eta_\rho + \eta_\sigma = 1$.

5 Discussions and Conclusions

The critical point $\alpha_\rho = \alpha_\sigma = 2$ (equivalently $\eta_\rho = \eta_\sigma = 1/2$), where the action for the model becomes that of two copies of the critical Schmid models, is found to be on the critical line (critical line A),

$$\frac{1}{\alpha_\rho} + \frac{1}{\alpha_\sigma} = 1, \quad \text{equivalently} \quad \eta_\rho + \eta_\sigma = 1, \quad (78)$$

as expected. However, it is not on the critical line $\eta_\rho + \eta_\sigma = 2$ which asserted in the previous work. The critical line A Eq.(78) (phase boundary) differs from the critical line (critical line B) discussed in refs.[9, 11]

$$\frac{1}{\alpha_\rho} + \frac{1}{\alpha_\sigma} = 2, \quad \text{equivalently} \quad \eta_\rho + \eta_\sigma = 2. \quad (79)$$

Let us choose a point $\alpha_\rho = \alpha_\sigma = 1$ ($\eta_\rho = \eta_\sigma = 1$) on the critical line B and examine the criticality of this point. At this point the action for the model is given as

$$\begin{aligned} S = & \frac{1}{4\pi} \int d\tau d\sigma [(\partial_\tau \theta)^2 + (\partial_\sigma \theta)^2] + \frac{1}{4\pi} \int d\tau d\sigma [(\partial_\tau \phi)^2 + (\partial_\sigma \phi)^2] \\ & + 2V_0 \int \frac{d\sigma}{2\pi} \cos \theta \cos \phi \Big|_{\tau=0}. \end{aligned} \quad (80)$$

In terms of the two boson fields $\phi_1 = \theta + \phi$, $\phi_2 = \theta - \phi$, the action can be written as

$$S = \frac{1}{4\pi} \int d\tau d\sigma \frac{1}{2} \sum_{a=1}^2 \partial \phi_a \partial \phi_a + V_0 \int \frac{d\sigma}{2\pi} \sum_{a=1}^2 (e^{i\phi_a} + e^{-i\phi_a}) \Big|_{\tau=0}. \quad (81)$$

If we scale the boson fields ϕ_a as $\phi_a \rightarrow \sqrt{2}\phi_a$, $a = 1, 2$,

$$S = \frac{1}{4\pi} \int d\tau d\sigma \sum_{a=1}^2 \partial \phi_a \partial \phi_a + V_0 \int \frac{d\sigma}{2\pi} \sum_{a=1}^2 (e^{i\sqrt{2}\phi_a} + e^{-i\sqrt{2}\phi_a}) \Big|_{\tau=0}. \quad (82)$$

By applying the Fermi-Bose equivalence, we may attempt to map the action of two boson fields ϕ_a , $a = 1, 2$ onto a Dirac fermion action of two flavors. But the boundary operator $: e^{\pm i\sqrt{2}\phi_a} :$ has a wrong dimension to be on the boundary. Although in the bulk it has a conformal dimension $(1/2, 1/2)$ so that it may be written as a fermion mass operator $\bar{\psi}^a \psi^a$, it has a dimension $(1, 1)$ thanks to the Neumann boundary condition, which doubles its dimension on the boundary. Hence, its dimension does not match that of the fermion bilinear. (See more detailed discussion on this point refs.[14, 19, 20].) We may rephrase this point in a slightly different way. We may write the operator $: e^{i\sqrt{2}\phi_a} :=: e^{i\sqrt{2}\phi_{aL}} :: e^{i\sqrt{2}\phi_{aR}} :$ as a fermion bilinear $\psi_L^{a\dagger} \psi_R^a$, but on the boundary with help of the Neumann boundary condition, it may be also written as $-i\psi_L^{a\dagger} \psi_L^{a\dagger}$, which is a null operator. That is, we cannot put the operator $: e^{i\sqrt{2}\phi_a} :$ as fermion bilinear field operators in a consistent way

on the boundary where the Neumann condition is imposed. Thus, the point $\alpha_\rho = \alpha_\sigma = 1$ ($\eta_\rho = \eta_\sigma = 1$) on the line B cannot be the critical point.

We should also note that it is easy to miss the additional contribution to the kinetic action of the boson fields θ_{ai} from the $U_A(1)$ chiral anomaly in the framework of the boson theory. It appears only through the non-trivial transformation of the path integral measure of the fermi fields under the $U_A(1)$ chiral phase transformation. Let us suppose that the additional contribution to the action of θ_{ia} from the chiral anomaly is missed. From Eq.(57,61), we see it is equivalent to replacing κ_1^2 and κ_2^2 as $\kappa_1^2 \rightarrow \frac{\alpha_\rho - 2}{4}$, $\kappa_2^2 \rightarrow \frac{\alpha_\sigma - 2}{4}$: Then the RG equation would be read as

$$\mathbf{G}_{\alpha\beta}^{(0)}[n] + \mathbf{G}_{\alpha\beta}^{(1)}[n] = \frac{V_0^2}{16\pi^2} \left(1 + \frac{1}{2} \left(\frac{\alpha_\rho}{4} + \frac{\alpha_\sigma}{4} - 1 \right) \zeta(1) + \cdots \right) G_{\alpha\beta}[n], \quad (83)$$

and the critical line would be obtained as

$$\alpha_\rho + \alpha_\sigma = 4, \quad (84)$$

which is certainly erroneous. Thus, the $U(1)$ chiral anomaly plays an important role to fix the critical lines of the model.

We conclude this paper with a remark on the renormalization of composite operators. Although the composite operators like four fermi boundary interaction terms, $\bar{\psi}_a \psi_a \bar{\psi}_b \psi_b$, $a = 1, 2$ cannot be included as a part of the bare boundary action, they may be generated as descendant operators in the high order radiative corrections. The renormalizability of the model at higher order may set more constraints on the parameters of the model, α_ρ and α_σ , bringing us richer structure of the phase diagram. The full structure of the phase diagram would be accomplished only by examining the renormalization of a complete set of the composite operators [26] at a given order. We will discuss renormalization of the composite operators along the direction of extension of this work elsewhere.

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